The March Calculation Policy

Progression from mental to written methods in the four operations

Introduction & Aims

This document outlines our agreed school policy and looks at the progression through the four operations. It outlines what and how we teach in order to make our teaching and learning consistent. This will give every child a clear written method of calculation for the four rules of number that they know how to use competently and confidently. It has also been devised with the requirements of Curriculum '14 in mind in that it moves pupils more rapidly to using a compact, vertical layout.

This policy aims to link the development of written methods with the practical resources, models and images and language used to support their teaching. This multi-faceted approach will promote an understanding of place value and the number system, enabling children to apply their knowledge when faced with an unfamiliar problem. They need to be given problem solving scenarios where they can use the formal written methods to find solutions. We believe that teaching one method for each of the four operations is far better than trying to teach too many methods. Flexible use of open number lines is also integral to teaching calculation at The March CEP particularly when supporting mental methods, solving problems involving negative numbers or time.

There is very clear progression throughout the year groups so that by the end of year six all pupils will have a formal written method for adding, subtracting, multiplying and dividing. This document outlines how the operations are taught within each key stage although teachers should be guided by pupil's individual development.

Everything we do within maths at The March CEP should be supported with visual imagery. It is of crucial importance that the teaching of written methods goes hand in hand with visual stimuli. Practical apparatus such as Numicon, rods and tracks, Multibase or homemade resources, e.g. bundles of straws, should be used to support all number work sessions and should be seen as a key feature in every classroom.

This policy has been developed and agreed by the whole staff. When calculation is taught throughout the school, the children will use the agreed algorithm, using practical apparatus to support, until the children feel that they are confident enough to work in the abstract alone. Children should not be moved on until they are ready. When progressing to the next stage it is crucial that the old stage is taught alongside it so that the children don't lose their confidence or misconceptions do not arise. They need to see how each layer is another step to making their understanding more absolute. If ever a child, or group of children, is unsure about how to do the more formal written method then they should go back to the stage before.

Mental Methods as a first resort.

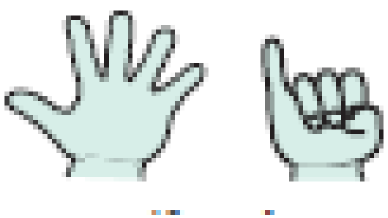
We need to be encouraging a very logical approach to finding the answers to problems, getting the children to use mental methods whenever they can. This policy details progression in written methods only and should be referred to **alongside** the benchmarks in mental maths. Use of the open number line is still very relevant but primarily to support mental calculations rather than a phase in the progression of written procedures. Every time they are confronted with a problem they should be asking themselves:

- **What do I think the answer will be** (Make a sensible estimate – what is the question asking?)
- **Can I do it in my head?**
- **Which practical apparatus should I use to help me?** (A choice needs to be available)
- **Can I do it with jottings?**
- **Should I use a number line?**
- **Shall I use a written method?**

Progression in Calculation – Addition

Emerging Addition EYFS

- As young children learn to count, early ideas of addition are explored through the idea of one more; *what is the next number? what number comes after…? What is one more than..?*
- The children then use *more than,* to compare two groups of objects initially then, once number recognition is secure, two numbers.
- Children are introduced to addition through practical problems; *How many altogether? How many more do we need?*
- *Three* ***and*** *two* ***is*** *five*, is used as a stepping stone to *Three* ***add*** *two* ***equals*** *five,* although no recording takes place at this stage other than child initiated jottings.
- Children are also taught addition as counting on using a number track. A natural means of introducing this is through board games although tracks with no numbers also have value in teaching children to count movements rather than spaces on the track. Links between these aspects of addition are made explicit, *Three* ***add*** *two* ***equals*** *five / Three* ***count on*** *two* ***equals*** *five.*
- *Recording addition begins with pictorial representation and is supported by practical work and visual imagery.*



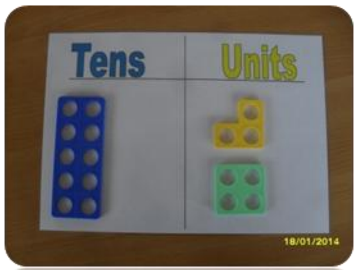
Progression in Written Addition KS1

1. Children need to have an understanding of the + and = signs. To understand the = sign, work on equivalencies and balances needs to be explored. It is essential children have a secure grasp of this symbol's meaning as an expression of equivalence rather than *the total* or *the answer.* All variations of simple additions should be explored at this stage.

4+1=5   5=4+1   4+□=5   □+1=5   □=4+1   5=4+□   5=□+1

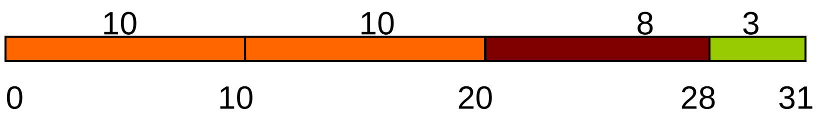
2. Horizontal layout is used for U + U. Once children progress to TU + U a vertical layout is introduced. Children are supported to partition two-digit numbers into tens and units using a simple grid. This begins without exchange progressing to TU + U, then TU + TU with exchange once place value is secure.

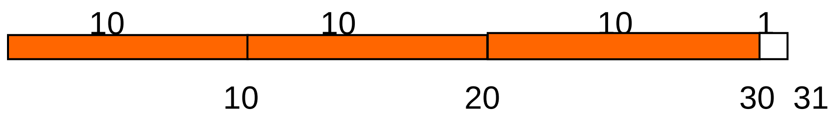
$$\begin{array}{r} 13 \\ +\ \ 4 \\ \hline \end{array}$$



$$\begin{array}{r} 18 \\ +\ 13 \\ \hline \end{array}$$

Number rods could also be used to support mental methods
18 + 13 = 31



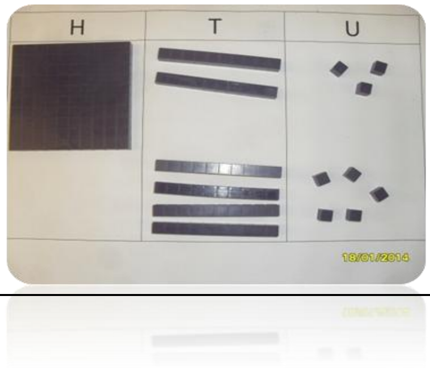


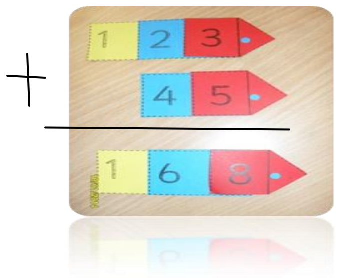
Progression in Written Addition KS2

3. Early ideas of partitioning into tens and units and recombining are now extended in the following order;

- HTU + U where the unit total does not exceed 9,
- HTU + U where the unit total does exceed 9,
- HTU + U where the unit total exceeds 9 and has a knock on effect making the total of tens exceed 9,
- HTU + TU where the total of tens does not exceed 9 tens,
- HTU + TU where the total of tens exceeds 9 tens,
- HTU + TU where exchange occurs in both the tens and units,
- HTU + HTU where exchange occurs.

In addition to Numicon and number rods, Dienes or place value counters are used. Arrow cards are also useful to bridge the gap between practical and numerical representation of addition. Methods can be expanded if needed.





HTU	HTU
123	1 1
+ 45	278
168	+ 346
	624
	12 14

4. The compact algorithm is refined with carried digits recorded below the line. The language of *carry ten* or *carry one hundred* is used rather than *carry one.* If children experience difficulties, they need to go back a step and use practical resources for longer. Once children are secure with the procedure, it can be applied to larger numbers, numbers with mixed numbers of digits and to decimals.

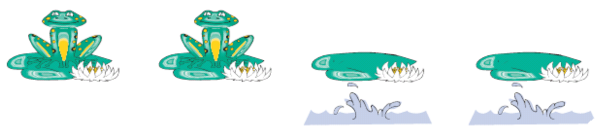
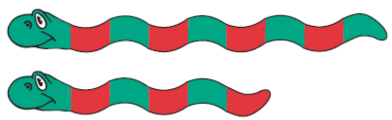
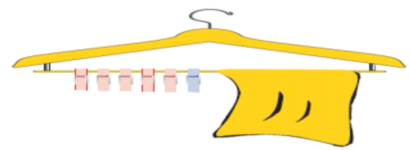
$\begin{array}{rcccc} & \text{Th} & \text{H} & \text{T} & \text{U} \\ & 7 & 6 & 4 & 8 \\ + & 1 & 4 & 8 & 6 \\ \hline & 9 & 1 & 3 & 4 \\ \hline & 1 & 1 & 1 & \end{array}$	$\begin{array}{rcccc} & \text{Th} & \text{H} & \text{T} & \text{U} \\ & 4 & 8 & 7 & 6 \\ + & & 6 & 2 & 5 \\ & & & 3 & 9 \\ \hline & 5 & 5 & 4 & 0 \\ \hline & 1 & 1 & 2 & \end{array}$	$\begin{array}{rccccc} & \text{T} & \text{U} & & \text{t} & \text{h} \\ & 2 & 4 & . & 6 & 3 \\ + & 3 & 8 & . & 5 & 9 \\ \hline & 6 & 3 & . & 2 & 2 \\ \hline & 1 & 1 & & 1 & \end{array}$

Nine and five and six equal twenty.
Put zero down, carry twenty.
Twenty and thirty and twenty and seventy equal one hundred and forty.
Put forty down, carry one hundred.
One hundred, six hundred and eight hundred equal fifteen hundred.
That’s one thousand, five hundred.
Put five hundred down, carry one thousand.
One thousand and four thousand equal five thousand.

Progression in Calculation – Subtraction

Emerging Subtraction EYFS

- Young children gain early practical experience of subtraction through counting games and rhymes, e.g. Five currant buns. They begin to relate the practical task of removing, e.g. a currant bun to finding one less. *How many are left? What number comes before…? What is one less than..?*
- The children then use *less than* to compare two groups of objects initially then, once number recognition is secure, two numbers.
- Children are introduced to subtraction through exploration of two of its complex structures, partitioning and comparison, (taking away and finding the difference).
- Taking away (the partitioning structure of subtraction) – children remove objects or pictures and count what is left. This can be supported pictorially by rubbing or crossing out. As with addition, a number track is used to relate removal of objects with counting back. *Five **take away** two equals three, Five **count back** two equals three, Two **less than** five equals three.*
- Finding the difference (the comparison structure of subtraction) – we encourage children to count on to find out how many steps or counts there are between two numbers? *How many more smilies do you need to get five? How many more children have a dog than a cat? How many less children have a fish than a rabbit?* Again the number track provides a valuable visual aid as children can see how big the gap is between two numbers.
- Using visual prompts, children are also exposed to subtraction as the inverse of addition during activities where part of a set is hidden, *There are seven eggs, you can see four, how many are hiding?*

<u>**Progression in Written Subtraction** KS1</u>

1. When the symbol for subtraction has been introduced this is first alongside a pictorial representation.

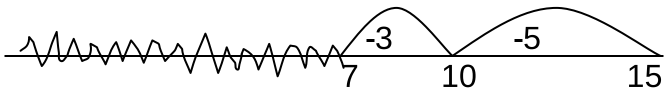
4 – 1 = 3

All variations of simple subtractions should be explored at this stage.

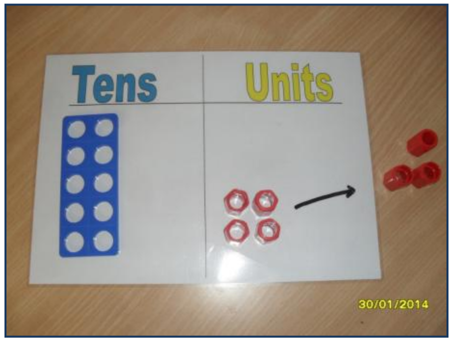
4 -1=□ □=4 -1 1=4 -□ 4 -□=1

2. Horizontal layout is used for U – U. Once children progress to TU – U a vertical layout is introduced although for finding the difference problems and until children become secure with exchange, a numberline is more suitable for recording. Both should therefore be used and children helped to understand which method is more appropriate. Subtraction by complementary addition is reserved principally as a mental method.

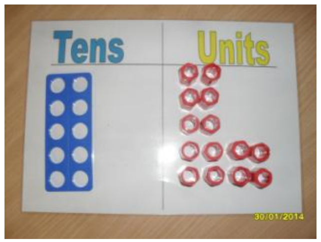
15 – 8 = 7

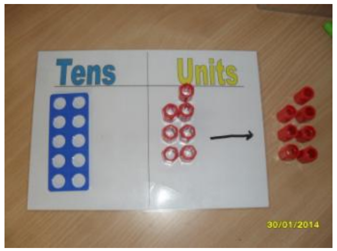


17 – 3 = 14



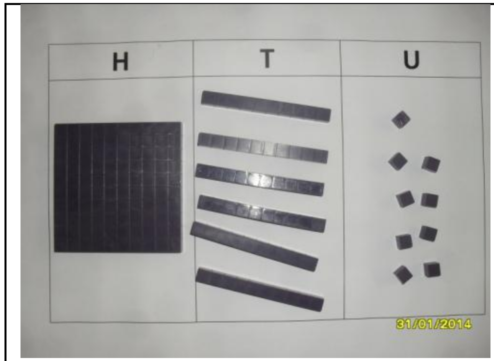
24 – 7 = 17 (with exchange or stealing a ten.)

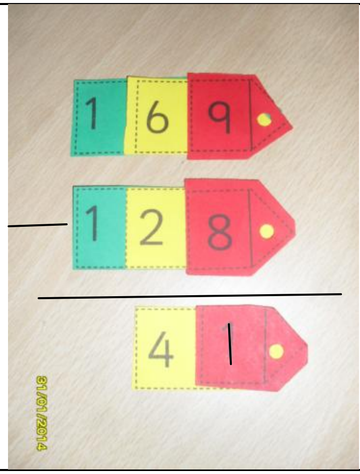




Progression in Written Subtraction KS2

3. Early ideas of exchange are now extended to match the abilities of the children in terms of the number of digits and degree of exchange necessary. The term *exchange* is more accurate as *borrowed* items are usually returned. We use the expression, *stealing a ten etc.* to help the children understand the procedure. In a similar way to addition, practical means of working through subtractions are first represented using digit cards and then numerically. Methods can be expanded to suit needs.






```
   H T U
   1 6 9
 - 1 2 8
     4 1
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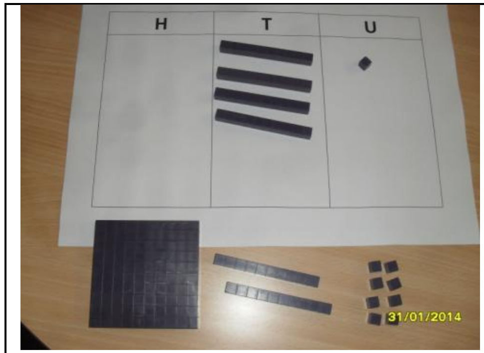
H T U 31/01/2014		

4. The compact algorithm is applied to subtractions with larger numbers, numbers with zeros as place holders and to decimals when children are ready. The language of *steal a ten* or *steal a hundred* is used rather than *steal a one.*

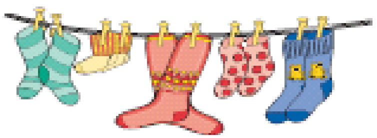
ThHT U 1 1 5 4 ~~2~~ 0 - 4 2 1 3 1 2 0 7	ThHT U 0 9 9 1 ~~1~~ ~~0~~ ~~0~~ 3 - 8 5 4 1 4 9	T U t h 2 1 5 1 £~~3~~ 4.~~6~~ 2 -£1 9.5 9 £1 5.0 3

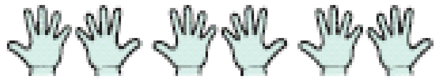
You cannot subtract three from zero. (Without going into negative numbers!)
Steal a ten.
Ten subtract three equals seven.
Ten subtract ten equals zero.
Four hundred subtract two hundred equals two hundred.
Five thousand subtract four thousand equals one thousand.

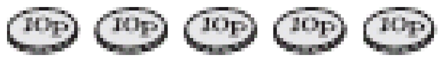
Progression in Calculation – Multiplication

Emerging Multiplication EYFS

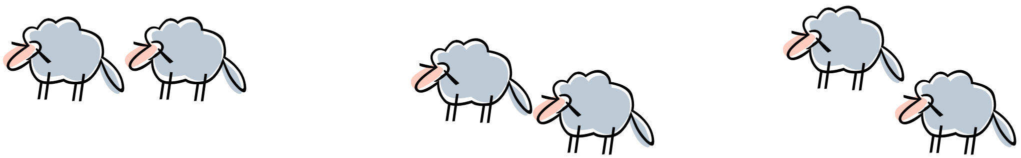
- Through practical means, children experience counting in twos initially, then tens and fives. This is reinforced by rote learning supported by objects or images, e.g. pairs of shoes or fingers.

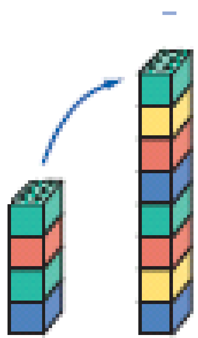






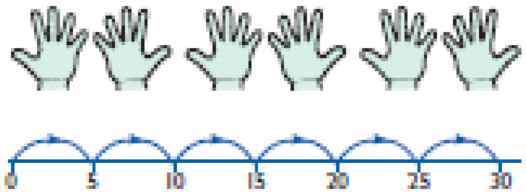
- Children link counting in multiples of a number to what they already know about patterns and the concept of a number pattern is formed.
- Links, such as the pattern of twos and doubling numbers and odd and even numbers are also explored. Doubling is a good way of introducing the scaling model of multiplication.
- The idea of grouping is introduced along with the language *three groups of two sheep.* (Aggregation structure of multiplication.)

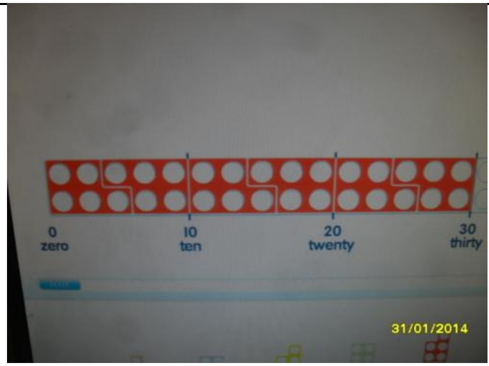


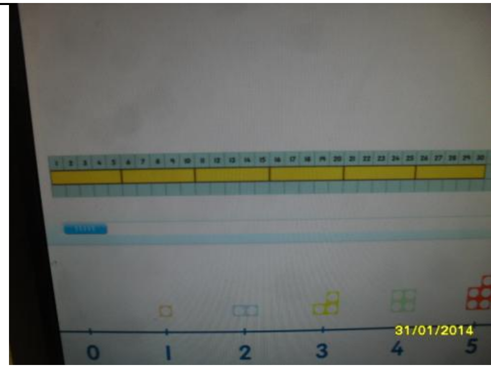


Progression in Written Multiplication KS1

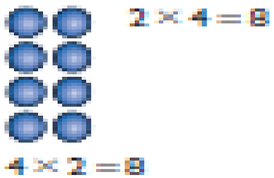
1. Children are supported to understand the vocabulary associated with multiplication as repeated addition. *Five added together six times, 5+5+5+5+5+5, six groups of five, six hops of five, six times five or five multiplied by six, 5x6.* Numicon and number rods provide a good image and can be used on the IWBs.

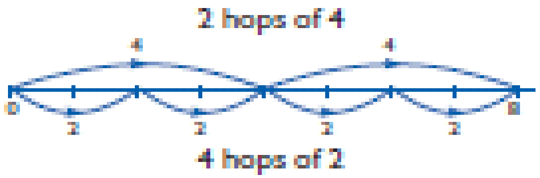






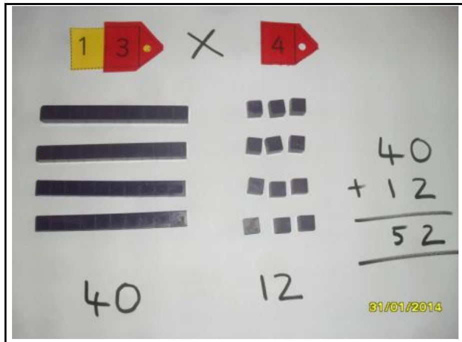
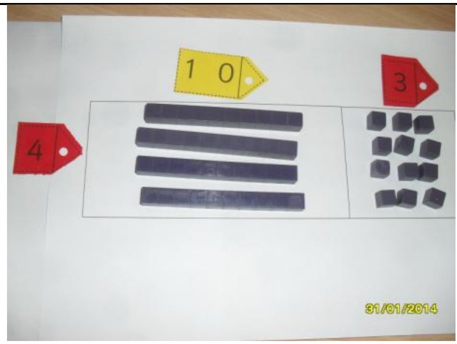
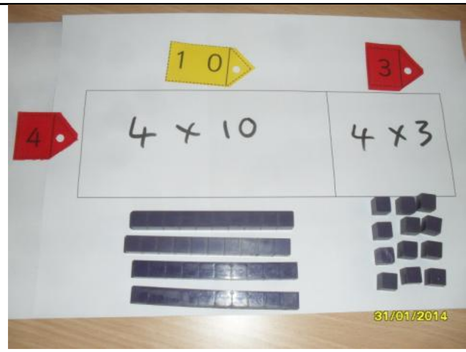
2. Arrays are then introduced which children describe using number sentences. Children understand that multiplication can be done in any order. They also clearly show the early links between multiplication and division.





Progression in Written Multiplication KS2

3. Early ideas of partitioning into tens and units and recombining are now extended using grid multiplication.
   The following stages show how this can be broken down.
   13 x 4 = 52

 Arrays with TU & U.	 Introducing a grid. Additions should be recorded vertically beside the grid if necessary.	 Writing each separate calculation in the grid.	$\begin{array}{c\|c\|c\|} \times & 10 & 3 \\ \hline 4 & 40 & 12 \\ \hline \end{array}$ $\begin{array}{r} 40 \\ +\underline{12} \\ \underline{52} \end{array}$ Using the grid.

4. Short multiplication is introduced alongside grid for TU x U and HTU x U.

x	300	40	2
7	2100	280	14

2100
280
+ 14
2394

3 4 2
x 7
2 3 9 4
2 1

Long multiplication is introduced for TU x TU, HTU x TU and ThHTU x TU and can be applied to decimals for those exceeding Y6 expectations. Naturally, children experiencing difficulties should refer back to the grid or expand the method to reinforce understanding.

x	20	4
10	200	40
6	120	24

200
120
+ 40
24
384

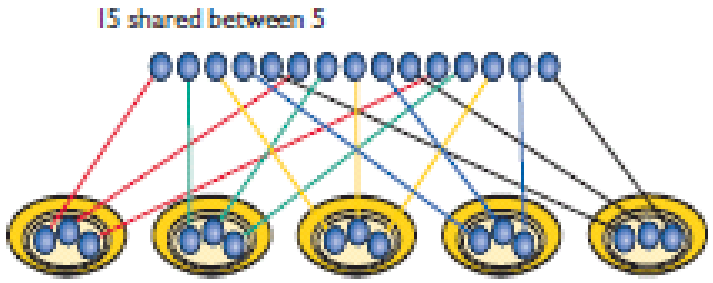
24
x 16
24(6x4)
120(6x20)
40(10x4)
200(10x20)
384

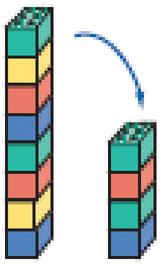
2 4
x 1 6
1 2 4 4
2 4 0
3 8 4

Progression in Calculation – Division

Emerging Division EYFS

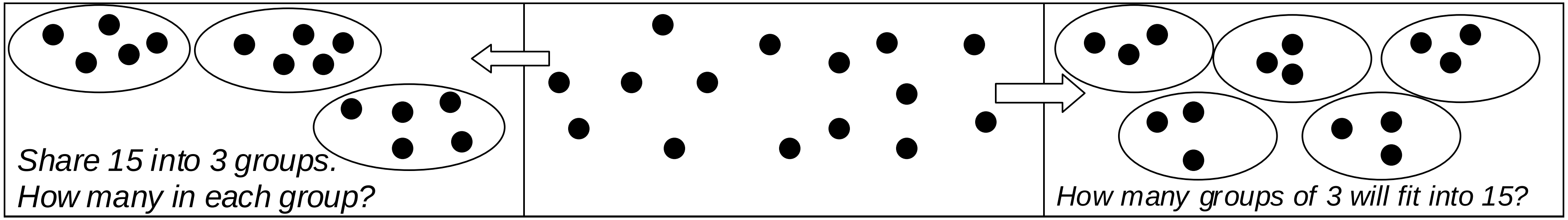
- Helping children to understand that there are two structures for division will lay firm foundations for the future. Young children relate very easily to the notion of division as sharing but the idea of division as grouping needs much exploration. In later years, this model of division becomes the inverse of multiplication and forms the basis of written calculations.
- Naturally through role-play young children share items, e.g. cakes in a basket between two plates. They discover that the outcome is not always equal and discuss ways in which this could be resolved. They learn that when things are shared equally between two this is called halving and the link with doubling is made explicit.
- Any recording at this stage would be done informally using pictures. Visual images can be helpful but asking children to record in the way illustrated below often leads to errors.



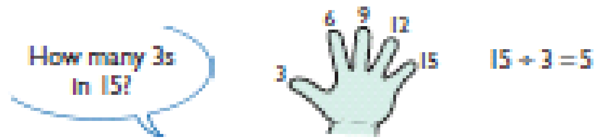


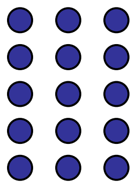
<u>**Progression in Written Division** KS1</u>

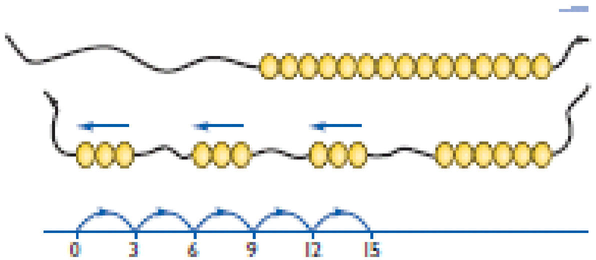
1. As children become more confident with multiplication, division as grouping, repeated subtraction or the inverse of multiplication can be introduced alongside. Explain that sharing larger numbers is hard to do so we use the inverse method instead as it is easier. Talk to the children about how they can use their knowledge of multiplication to solve division problems.

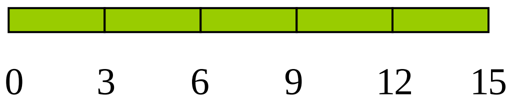


2. They will already be familiar with arrays but other practical resources can be used to support children to understand the meaning of, e.g. 15 ÷ 3 = 5, including bead strings, Numicon, number rods and number lines.



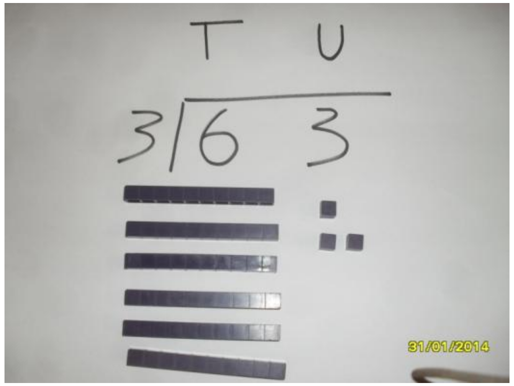


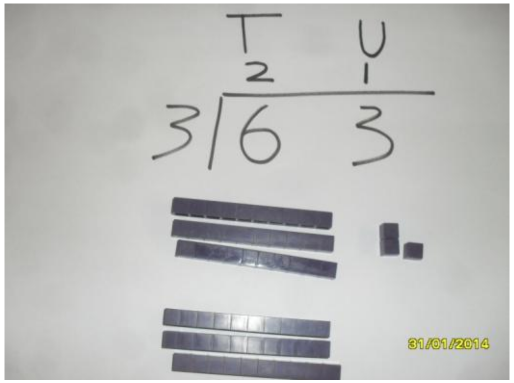




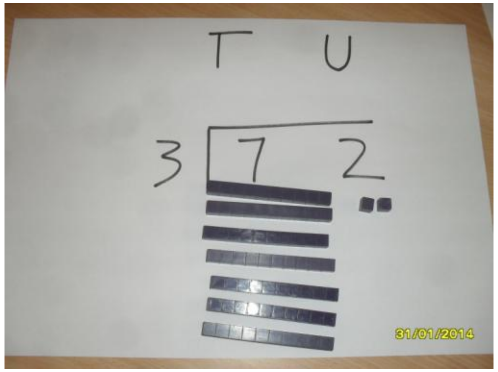
Progression in Written Division KS2

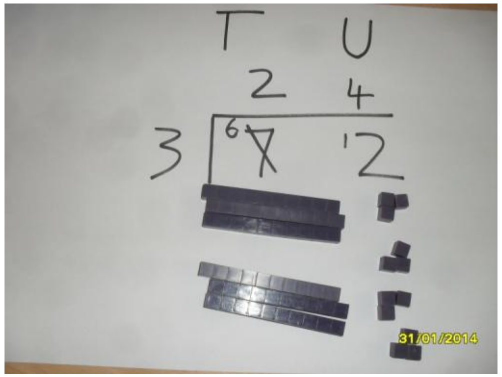
3. Short division is introduced in carefully structured steps, TU by U with no exchange, TU by U with exchange followed by HTU by U in a similar way. Division with remainders is not expected until Year 5. Numicon can be used very effectively to show fractional remainders.



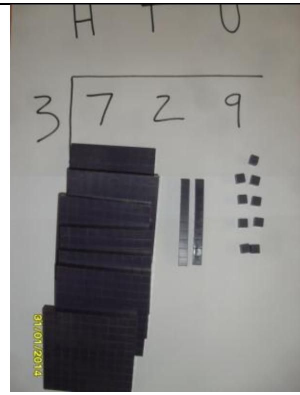


TU by U with no exchange.





TU by U with exchange.



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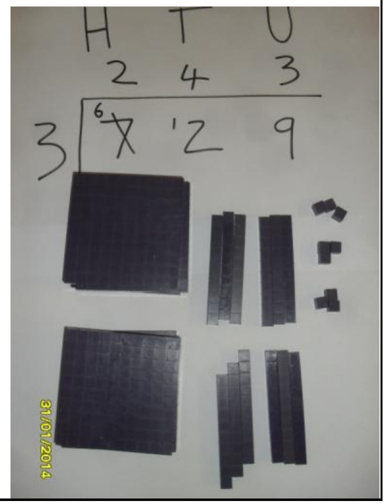
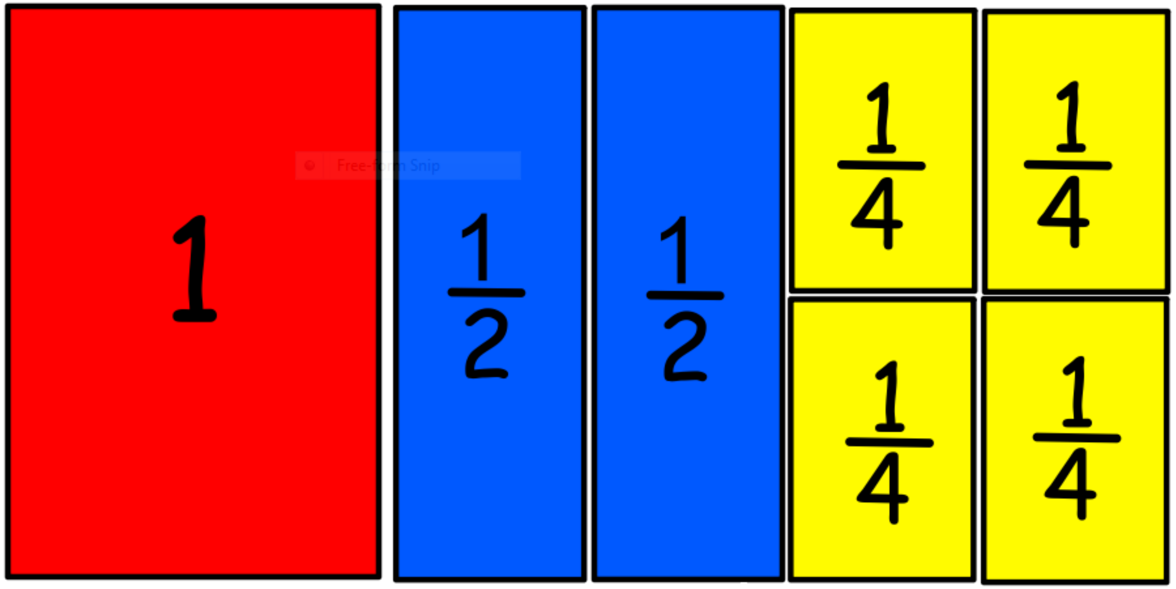


4. Long division comes in for dividing numbers up to 4 digits by a two-digit number. Remainders can be interpreted as whole number remainders, fractions, decimals or by rounding dependent on context. Short division can also be used where appropriate, again interpreting remainders according to the context.

4 5 r 1 5 **1 1 | 4 9 6** Answer: $45\frac{1}{11}$	**2 8 r 12** **1 5 | 4 3 2** **3 0 0** **1 3 2** **1 2 0** **1 2**	Menu method expands this out. 10 x 15 = 150 20 x 15 = 300 2 x 15 = 30 4 x 15 = 60 8 x 15 = 120
2 8 **1 5 | 4 3 2** **3 0 0** **1 3 2** **1 2 0** **1 2** $\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$ Answer: $28\frac{4}{5}$	**2 8 · 8** **1 5 | 4 3 2 · 0** **3 0** **1 3 2** **1 2 0** **1 2 0** **1 2 0** **0** Answer: 28·8	

Progression in Calculation using fractions KS1

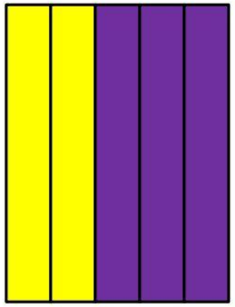
1. Children recognise, find, name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity. There is no requirement to calculate using fractions. However, provided tasks are practical and provide easily accessible representations of the mathematics involved, simple addition and subtraction of fractions involving wholes, halves and quarters could be introduced as extension or investigation.



<u>Progression in Calculation using fractions</u> KS2

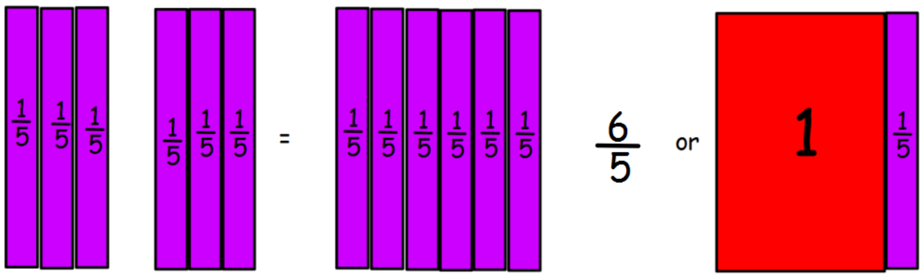
2. In Lower KS2, children add and subtract fractions with the same denominator (restricted to within one whole in Y3). This can be represented effectively in the following way.

$$\frac{2}{5} + \frac{3}{5}$$



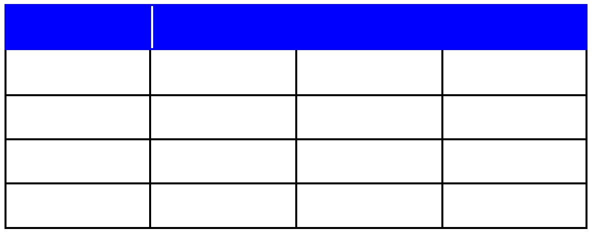
3. In Year 5, children multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.

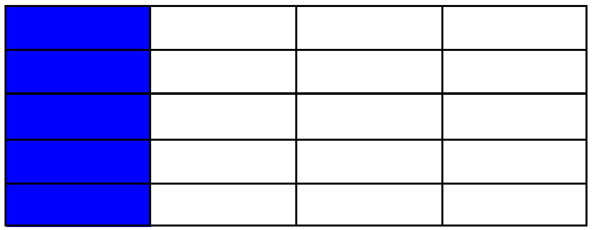
$\frac{3}{5} \times 2$



4. In Year 6, the expectation rises again. Children must add and subtract fractions with different denominators and mixed numbers using the concept of equivalent fractions. This will have been covered in Year 5.

1/5 + ¼



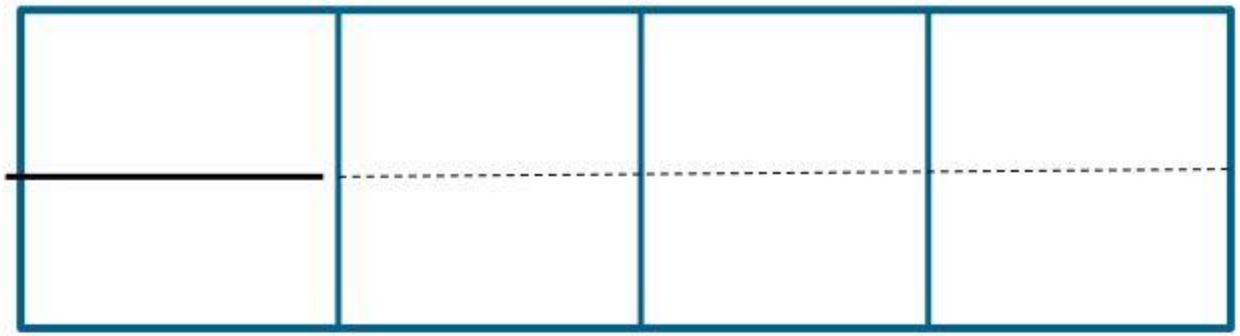


1/5 + ¼ = 9/20

5. Children also encounter the multiplication and division of fractions by fractions.

- Multiply simple pairs of proper fractions, writing the answer in its simplest form.

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$



- Divide proper fractions by whole numbers.

The link between, e.g. dividing by 2 and multiplying by ½ needs to be made.

$$\frac{1}{3} \div 2 = \frac{1}{6}$$